Necessary Existents and State Space Theory

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Necessitism is a controversial view concerning the layout of the modal landscape. It seeks to settle the question of which things exist in the different possibilities for how our universe could have unfolded. There are good reasons to take necessitism to be correct. In §1 I will outline the view being defended before, in §2, presenting an argument for why it should be accepted. In brief, our best scientific theories commit us to the truth of necessitism. But the consequences of the view remain controversial and I will explore two ways to push back against the argument. These are the topic of §§ 3-4 respectively. Both responses will be seen to be unsatisfactory so that the argument for necessitism from state space theory remains a compelling reason to adopt the view.

1 Necessitism

Trump could have failed to win the electoral college vote. As a result,

Donald Trump is currently president of the USA.

is only *contingently* true. The history of the world could have played out differently and resulted in a Clinton presidency in 2017. More dramatically, it seems that Trump's very existence is a contingent matter.

(2) Donald Trump might not have existed.

His parents, Fred and Mary Anne, might never have met. Or they might have angrily fallen out on being introduced to each other and never spoken again. Or the earth might have formed in an uninhabitable region of the solar system. Indeed, there appears to be countless ways in which the history of the universe could have played out so that Trump did not exist. It is instructive to contrast this case with that of mathematical objects, which are generally taken to exist *necessarily*. Unlike Trump, there is no possible way that the world could be according to which the number 7 did not exist. More mundane entities such as tables, chairs and presidents – unlike mathematical objects – seem to exist only contingently.

According to *necessitism*, this appearance of contingency is misleading. In a slogan, this view holds that

(N) It is necessarily the case that everything exists necessarily.[1]

For example, Trump, like the number 7, exists necessarily. There is no possible history of the world in which Trump fails to exist. Perhaps more counterintuitively,

[1] The relevant claim of modal logic is $\Box\forall x\Box\exists y\ x = y$. Necessitism has been defended by Linsky and Zalta (1994) and more recently by Williamson (2013).

necessitism is committed to the existence of *merely possible* objects. For instance, it is certainly possible that Clinton and Trump had a daughter, whom we may call 'Clump.' That is to say, there is a possible history of the world in which Clump exists. But if N is true, then Clump too exists necessarily. She exists in all the possible histories of the world, and in particular, she exists in the actual world. Or so say the necessitists.

Necessitists have argued that the most controversial consequences of their view, such as that Trump and Clump necessarily exist, withstand scrutiny. The central move is to distinguish between existence simpliciter and *concrete* existence. A concrete existent is an object that exists in space and time. Crucially, necessitists hold that an object can exist without being a concrete existent. An object might be a concrete existent in one possibility and a non-concrete existent in another. For instance, in the actual world Trump is a concrete existent, while in the possible world where his parents never met he is a non-concrete existent. Analogously, in the possible world where she is conceived by Clinton and Trump, Clump is a concrete existent but in the actual world she is a non-concrete existent. Trump and Clump both exist necessarily, since they exist in all possible worlds. But it is a contingent matter whether they are concrete or not.

2 Argument from state space theory

Distinguishing between concrete and non-concrete existence fails to amount to a compelling case for necessitism. Why should we complicate our understanding of what exists by allowing that Clump exists as a contingently non-concrete object? Why should we not just reject the need for the contingently non-concrete and insist that Clump *does not exist* in the actual world, although there are other possibilities in which she does exist? Necessitists have tried to motivate their view beyond these suggestive remarks in a range of ways.[2] Most recently, Williamson has tried to draw support from contemporary scientific practice by arguing that only necessitists can account for the successful use of state space theory. Necessitism, he claims, is presupposed by our best scientific theories.

State spaces appear throughout science and are used to formulate our best theories of the world. They appear in physics, to model the interactions of fundamental particles, as well as biology, where they can represent population changes, and economics, to describe the behaviour of stock prices. A state space consists of the set of all possible states of a physical system at a time.[3] Each state represents a possible way for the system to be. Typically, no path goes through every state in the space and hence some states represent *merely possible* ways for the system to be.

The central claim of the necessitist argument is that the theory of state spaces presupposes a necessitist account of what exists. Williamson claims that applications of state

[2] For instance, Williamson has argued that necessitism is superior to its denial, contingentism, on the grounds that necessitist modal logic is stronger than its rivals. The former, he claims, is simpler, less *ad hoc* and supports more true generalisations about metaphysical modality. Williamson (2013).

[3] A state space is made up of a set S with a topological or geometrical structure, an indexing set T with an additive structure, and a set of functions $\{f_t : t \in T\}$ such that for all $s \in S$

(i) $f_0(s) = s$

(ii) $f_t f_{t'}(s) = f_{t+t'}(s)$.

Williamson (2016a, p. 472), Nolte (2010, p. 33).

space theory in science 'implicitly take for granted a necessitist modal logic.'[4] In the systems studied in science, individuals are sometimes created or destroyed. For instance, biologists use state spaces to model ecosystems where the population varies and physicists use spaces to represent particles that are created and then annihilated. It may appear to be a contingent matter what individuals there are, since a given individual may be created on one trajectory through the system but not on another. However, state space theory involves generalising over all the states and sets of states, which is in effect to quantify over merely possible individuals. Scientists regularly quantify over objects such as merely possible predators and merely possible particles, which assumes a necessitist conception of what there is to quantify over. In order to formulate the best current theories of, say, particle interactions, it is necessary to refer to particles that are concrete in some possible states of the system, but non-concrete in the actual state. In brief, state space theory involves modal commitments that can only be accounted for within a necessitist framework.

From these considerations an argument for one of necessitism's most controversial claims, namely that there exist contingently non-concrete objects such as Clump, may be extracted.[5]

Contingently non-concrete objects are indispensable to our best scientific theories.
If contingently non-concrete objects are indispensable to our best scientific theories, then contingently non-concrete objects exist.
Therefore, contingently non-concrete objects exist

In order to test the strength of Williamson's argument from state space theory, I will consider two natural responses. These contingentist manoeuvres are unconvincing.[6]

3 Response I: eliminative contingentism

One response to this line of argument would be to accept Premise II but deny Premise I. This would involve accepting the existence of contingently non-concrete objects if they could be soon shown to be indispensable to the best current theories. But, this response continues, such objects are not in fact indispensable. Rather, necessitist commitments in scientific practice can be *eliminated*. This would allow contingently non-concrete objects of state space theory to be treated as useful fictions. Although these objects do not actually exist, they allow scientists to elegantly state and then fruitfully study our best scientific theories.[7] If contingently non-concrete objects are eliminable from the best current scientific theories, then scientists may use them as theoretical posits without being committed to their existence.

In order to show how contingently non-concrete objects are eliminable from state space theory revisions to the logic used there may be proposed. Contingentists can try to simulate in their own terms the necessitist effect of quantification using their own logical

[4] Williamson (2016b, p. 576).
[5] This form of argument is influential in the philosophy of mathematics, where a similar argument is taken to support mathematical platonism. Cf. Quine (1960, Chapter 7), (1969, p, 97) and Putnam (1971, p. 37).
[6] Contingentism is the denial of necessitism, namely the claim that some objects only possibly exist.
[7] Field (1980) defends this response to the Quine-Putnam argument in the philosophy of mathematics.

apparatus. Contingentist quantifiers range only over *actual* objects. Nevertheless, they allow contingentists to simulate the effect of necessitist quantification over contingently non-concrete objects.[8] Crucially, however, the reconstruction breaks down in the case of second-order quantification, where merely possible *properties* as well as merely possible objects are quantified over.[9] The necessitist account of second-order quantification cannot always be simulated in contingentist terms. Yet such quantification over properties of possible states is ubiquitous in the study of state spaces. Hence, revising the logical principles alone does not show how contingently non-concrete entities can be eliminated from the best current scientific theories.

It may be hoped that the contingentists response can be sustained if mathematical principles in state space theories are revised. One such principle central to the use of state spaces in analytic mechanics is Hamilton's Principle.[10] In the case of simple systems, there is a statement equivalent to Hamilton's Principle that does not mention possible histories of the relevant system.[11] This is precisely the kind of result needed to show that the *eliminative* response is viable. But for mechanical systems that are not simple, this suggestive equivalence breaks down. Moreover, there is no reason to suppose that the modal commitments can be eliminated in the more complex cases. While simple principles in analytic mechanics can arguably be expressed without necessitist commitments, attempts to remove these commitments from *all* the mathematical principles of the best current theories appear hopeless.

Even supposing that contingentists can extend their logic to circumvent these concerns, or put forward mathematical principles that are fully equivalent to those used in the best current theories, a methodological problem remains. Even granting for the sake of argument that it is possible to paraphrase state space theory so that it is in fact acceptable to the contingentist, this alone does not answer the indispensability argument of the previous section. For the most straightforwardly interpreted theory may be superior, on general theoretical grounds, to the paraphrased replacement. The proposed replacements are likely to be *ad hoc* and cumbersome, lacking the theoretical virtues of comparative simplicity and strength displayed by the current state space theory. Even if theories of state spaces are developed where reference to contingently non-concrete objects is eliminated, such theories are unlikely to constitute part of the best current scientific theories. Yet it is the best scientific theories, rather than more cumbersome alternatives, that are looked at in order to determine what really exists.

4 Response II: Non-eliminative contingentism

Given the difficulties faced by an attempt to eliminate contingently non-concrete objects from our best scientific theories, the contingentist might try to undermine the argument from a different direction. This would involve attempting to show that we need

[8] The idea is to understand 'there is a possible object *x*' as 'possibly there is an object *x*'. Fine (1977, p. 118)
[9] Williamson (2013, p. 305).
[10] Hamilton's Principle states that, in the case of simple state space systems, the action integral of the Lagrangian for the system is stationary for the actual path. The principle allows theorists to state the actual law as a condition that compares the actual history of the system with counterfactual histories that do not obey the law. Butterfield (2004, p. 14).
[11] Butterfield (2004, p. 29).

not be committed to the existence of contingently non-concrete objects, even granting that reference to them cannot be eliminated from the best current scientific theories. That is, this reply concedes Premise I, but denies Premise II.

When scientists investigate the state space of a physical system they often do so for the purpose of model-building. In so doing, simplified mathematical models of the target system are developed to help us understand its actual features. It may be that the assumption of merely possible particles or predators in applications of state space theory is an idealisation employed in order to make theoretical models tractable. As such, the assumption that merely possible objects exist should not be taken to be true just because it is employed in theoretical models. This would fit with aspects of scientific practice more generally. Scientists often make idealizations in their theories that are explicitly contradicted by other theoretical assumptions. For instance, in order to have a tractable theory of the dynamic behaviour of fluids, it is assumed that fluids are continuous substances, even though this is known to be false.[12] Likewise, it might be argued that even if it is *indispensable* to a tractable theory of analytic mechanics to quantify over contingently non-concrete objects, this does not commit us to their existence.

In order for this contingentist position to be viable, it must be shown how it is possible to coherently quantify over merely possible objects in the best scientific theory and then go on to deny that these objects exist. Suppose that the state space theory used by physicists implies

(P) There is a merely possible particle x.

Some philosophers have argued that in some cases one can legitimately assert a collection of sentences whilst denying some of their logical consequences.[13] Melia describes this as 'retracting an implication.' In general, sentences are asserted in order to present a picture of how the world is and normally think of each successive sentence as adding a further layer of detail. Melia argues that we can understand some of the later sentences as taking back things that we earlier claimed by erasing or changing some of the implications in the previous part of the theory. If this practice is coherent, then the contingentist may make full use of state space theory and then finally clarify 'but there are no such things as contingently non-concrete objects.'[14] The contingentist explains that the implications that the state space theory has for the existence of such things as contingently non-concrete particles are in fact false, but that the rest of the implications remain true.

This attempt to shirk the need to reconstruct scientific theories in order to eliminate the reference to contingently non-concrete objects cannot succeed. For, this proposal relies on there being a non-contradictory reading of

(P*) There is a contingently non-concrete particle x […] but there are no contingently non-concrete objects.

[12] Maddy (1997, p. 144) and Leng (2010, p. 112) make analogous moves in the philosophy of mathematics.
[13] Melia (2000, p. 456).
[14] Cf. Melia (2000, p. 467).

It is incumbent on the contingentist pursuing this position to explain why P* should not be understood as an obvious contradiction.[15] One way of doing so would be to provide an appropriate paraphrase of P in order to show that, properly understood, it does not have the implication that merely possible objects exist. Statements of the best theories can quite often be paraphrased, as when it is claimed 'I didn't mean what I claimed, what I really meant was …,' followed by the appropriate paraphrase. It is this way that the statements that are being retracted from the rest of the theory that is considered to be true are separated out.

However, if a paraphrase for something that has been claimed cannot be provided, then we are not in a position to retract the claim without contradicting ourselves.[16] In the absence of paraphrase therefore, contingentists cannot coherently deny that they are contradicting themselves in making assertions such as P*. Of course, this criticism would be disarmed if an appropriate paraphrase of P that would give a non-contradictory reading of P* could be provided. But this takes us back to the eliminative project, which there was no reason to take to be viable. A non-eliminative approach therefore fails to advance the contingentist position beyond its unpromising eliminative counterpart.

Conclusion

In the models of state space theory, different trajectories through a system correspond to different possible states for the system to be in. Some trajectories model merely possible states of the system, in which there are merely possible objects. In order to formulate the best current scientific theories, such merely possible objects must be quantified over and hence, if science is to be taken at face-value, we are ontologically committed to such objects. This shows that the necessitist picture of what exists is correct. Of course, the view remains counterintuitive since it shows that Donald Trump, for instance, is a necessary existent. He exists (non-concretely) even in possibilities where his parents never met. However, just as pre-theoretic intuitions should carry no weight against the implications of the best theories in physics, so pre-theoretic intuitions should not be relied on in the realm of metaphysics.

[15] Colyvan (2010, p. 295).
[16] Azzouni (2009).

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